Efficient ARMA Modeling of FDTD Time Sequences for Microwave Resonant Structures

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Abstract: The finite-difference time-domain (FDTD) method requires computation of very long time sequences (TS) to accurately characterize the slowly decaying transient tail of resonant and/or electrically large structures. Therefore, it becomes critical to investigate methods of reducing the computational time for such objects. In this paper, we present a new signal processing algorithm, which uses significantly lower model orders than those employed in existing Prony-based algorithms, to extrapolate the late-time signature accurately from the moderately early-time TS. The robustness and accuracy of the new method are illustrated by the FDTD simulation and experimental corroboration of a high-Q microstrip filter, an example of a resonant structure.

1. Introduction

The FDTD technique is well-established as a versatile analytical tool for solving EM problems associated with characterizing microwave structures. However, it is also recognized that for accurate characterization of the underlying modes of even relatively simple structures, very long time-sequences need to be computed. The problem becomes particularly acute for high-Q structures with energy-storage features such as stubs and cavities, which tend to manifest relatively long decay times requiring thousands of time steps. In order to avoid such long computation times and associated cost, several researchers have recently attempted to model the TS from short 'early' data records and then use the estimated (or trained) 'model' to predict the remainder of the TS by extrapolation (*cf.* [1], [2]).

It has been argued that the FDTD-TS can be modeled as the Impulse Response (IR) of an Auto Regressive Moving Average (ARMA) transfer function. However, it is known that determination of ARMA parameters by IR fitting is a non-linear optimization problem. In most of the current EM literature Prony's method and its modifications appear to play a dominant role. This may be due to the fact that Prony's method converts the essentially non-linear modeling problem into a relatively simple linear estimation problem. However, Prony's approach suffers from two drawbacks. First, it tends to overmodel the system. In particular, when there are deep nulls in the frequency domain, significantly high model orders are needed to achieve a good fit. Secondly, overmodeling may lead to instability. In order to circumvent these problems, we propose to minimize the true non-linear error criterion using an efficient iterative method for signal extrapolation, which significantly improves IR fitting with relatively few ARMA model parameters (*i.e.*, low model orders).

In the proposed approach, the model parameters are estimated by minimizing the true fitting error (FE) criterion which is non-linear; hence, very good fit to the FDTD time-sequence and accurate frequency domain response can be achieved even with significantly low model orders. In contrast, the Prony-based linear predictor-type methods used in most EM literature minimize a simpler, but entirely different, 'equation error' (EE) criterion that only approximates the true FE criterion. As a result, significantly high model orders are needed in these methods to achieve good approximation in the frequency-domain [1], [2].

2. Problem Statement

A general complex exponential model of the FDTD signal can be defined as the impulse response of a linear time-invariant system,

$$h(n) \triangleq \sum_{k=1}^{p} \alpha_k e^{(\sigma_k \; + \; j\omega_k)n + j\phi_k}, \quad n = 0, 1, 2, \ldots \quad (1)$$

where α_k, σ_k, ω_k and ϕ_k denote the real amplitude, damping factor, frequency and initial phase, respectively, of the k-th exponential. However, the time-sequences generated by FDTD simulations are always

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real and consequently, they can be represented as

$$h(n) \triangleq \sum_{k=1}^{\frac{p}{2}} \alpha_k e^{\sigma_k} \cos(\omega_k n + \phi_k), \quad n = 0, 1, 2, \ldots \quad (2)$$

Note that p has been assumed to be even without any loss of generality. Taking z-transform of $h(n)$ in (1),

$$H(z) = \sum_{k=1}^{p} \frac{A_k}{(1 - d_k z^{-1})}, \quad (3)$$

where, $A_k \triangleq \alpha_k e^{j\phi_k}$, denotes complex amplitudes and $d_k \triangleq \sigma_k + j\omega_k$. After summation of the p terms in the right hand side, we obtain

$$H(z) = \frac{a_0 + a_1 z^{-1} + \cdots + a_{p-1} z^{-(p-1)}}{1 + b_1 z^{-1} + \cdots + b_p z^{-p}} \triangleq \frac{N(z)}{D(z)}, \quad (4)$$

where the coefficient of z^0 term in denominator has been assumed to be unity. Note that all coefficients in this transfer function (TF) are real because all A_k and d_k occur in conjugate pairs. It may also be noted that a general ARMA TF is given by

$$H(z) = \frac{a_0 + a_1 z^{-1} + \cdots + a_{q-1} z^{-(q-1)} + a_q z^{-q}}{1 + b_1 z^{-1} + \cdots + b_{p-1} z^{-(p-1)} + b_p z^{-p}}, \quad (5)$$

where p and q denote the number of poles and zeros, respectively, and can have any arbitrary integer values. With $p = q + 1$, the special ARMA TF in (4) is known as the 'strictly proper' case. Another important special case is the purely Auto Regressive (AR) TF with $q = 0$. The transfer function $H(z)$ in (5) can be written equivalently in terms of its impulse response as

$$H(z) = h(0) + h(1)z^{-1} + \cdots + h(k)z^{-(k)} + \cdots \quad (6)$$

Stacking the first N 'significant' samples of $H(z)$,

$$\mathbf{h} \triangleq [h(0) \quad h(1) \quad \cdots \quad h(N-1)]^T. \quad (7)$$

Next, let the vector containing the N samples of the measured FDTD-TS be denoted as

$$\mathbf{h}_d \triangleq [h_d(0) \quad h_d(1) \quad \cdots \quad h_d(N-1)]^T. \quad (8)$$

According to Steiglitz [3], given a desired impulse response $\mathbf{h}_d$, 'the ideal problem' of optimal estimation of the parameters a_i and b_i can be represented by the following FE minimization,

$$\begin{aligned} \min_{\mathbf{a},\mathbf{b}} ||\mathbf{e}||^2 &\triangleq \min_{\mathbf{a},\mathbf{b}} \sum_{i=0}^{N-1} \left[h_d(i) - \left\{\frac{N(z)}{D(z)}\right\}\delta(i)\right]^2, \\ &\triangleq \min_{\mathbf{a},\mathbf{b}} ||\mathbf{h}_d - \mathbf{h}||^2 \quad (9) \end{aligned}$$

$$\begin{aligned} \delta(i) &= \begin{cases} 1, & i = 0 \\ 0, & i \neq 0 \end{cases} \\ \mathbf{a} &\triangleq [a_0 \quad a_1 \quad \cdots \quad a_q]^T \\ \mathbf{b} &\triangleq [1 \quad b_1 \quad \cdots \quad b_p]^T. \quad (10) \end{aligned}$$

The notation, $\left\{\frac{N(z)}{D(z)}\right\}\delta(i)$ denotes the i-th output when a 'system' represented by $\frac{N(z)}{D(z)}$ is driven by the input $\delta(i)$. This problem is known to be non-linear in $\mathbf{b}$ and standard non-linear optimization algorithms have been suggested [4], [5].

3. Signal Processing Approach

Recently, the joint FE optimization problem defined in (9) has been theoretically *decoupled* into two subproblems of *reduced* computational complexities by Shaw [6]. The approach, known as decoupled Optimal Method (OM), is applicable to identification of rational models with *arbitrary* numbers of poles (p) and zeros (q). It has been shown that the non-linear denominator subproblem possesses a weighted-quadratic structure which can be utilized to formulate an efficient *iterative* minimization algorithm. It has also been established in [6] that the decoupled sub-criteria of OM possess *global optimality* properties. A brief outline of OM follows.

The non-linear denominator criterion of OM has the following form:

$$\begin{aligned} \min_{\mathbf{a},\mathbf{b}} ||\mathbf{e}_{OM}(\mathbf{a},\mathbf{b})||^2 &\equiv \min_{\mathbf{b}} ||\mathbf{e}(\mathbf{b})||^2 \\ &\triangleq \min_{\mathbf{b}} ||\mathbf{B}(\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{h}_d||^2 \\ &= \min_{\mathbf{b}} ||\mathbf{h}_d^T\mathbf{B}(\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{h}_d||^2 \\ &= \min_{\mathbf{b}} \mathbf{b}^T\mathbf{H}_2^T(\mathbf{B}^T\mathbf{B})^{-1}\mathbf{H}_2\mathbf{b} \end{aligned} \quad (11)$$

where

$$\mathbf{B} \triangleq \begin{bmatrix} b_{q+1} & b_{q+2} & \cdots & b_p & \cdots & 0 \\ b_q & b_{q+1} & \cdots & b_{p-1} & \ddots & \cdots \\ \vdots & \vdots & \ddots & \vdots & \ddots & b_p \\ 1 & b_1 & \cdots & \cdots & \cdots & b_{p-1} \\ 0 & 1 & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & b_1 & \cdots & \cdots \\ \cdots & \cdots & \cdots & 1 & \cdots & \cdots \\ 0 & 0 & \cdots & \cdots & \cdots & b_1 \\ 0 & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}$$

and the $(N - q - 1) \times (p + 1)$ matrix $\mathbf{H}_2$ is formed as, $H_2(i, j) = h_d(i - j + q + 1)$. Once $\mathbf{b}$ is estimated by optimizing (11), the numerator is found by the following pseudo-inverse solution

$$\mathbf{a} = \mathbf{H}_2^{\#}\mathbf{h}_d \quad (12)$$

Note that, in deriving (11) and (12), no linearization had been used in [6].

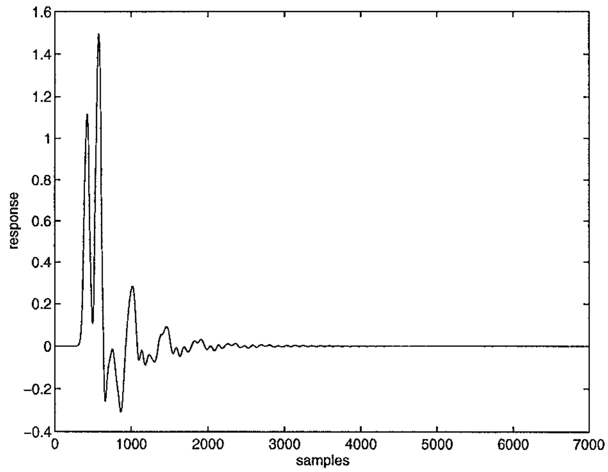

Figure 1: The raw FDTD TS of a microstrip low-pass filter.

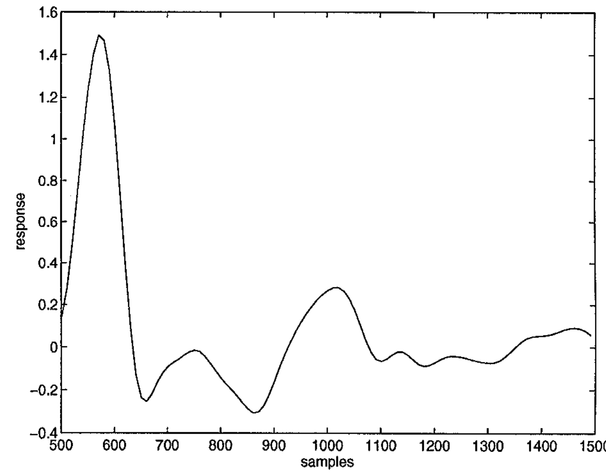

Figure 2: Decimated FDTD TS of the filter.

The criterion in (11) is non-linear in $\mathbf{b}$, but it possesses a weighted-quadratic structure where the weight matrix itself depends on the unknown parameters in $\mathbf{b}$. An iterative minimization scheme is given in [6], where the initial estimate of $\mathbf{b}$ is found by setting the 'prefilter' matrix $(\mathbf{B}^T\mathbf{B})^{-1} = \mathbf{I}_{(N-q-1)}$ in (11), *i.e,* by optimizing,

$$\min_{\mathbf{b}} \mathbf{b}^T\mathbf{H}_2^T\mathbf{H}_2\mathbf{b}. \tag{13}$$

Interestingly, this criterion is identical to the so-called 'covariance method' of linear prediction used in all Prony-based methods reported in EM literature. Clearly, since the Prony's estimator is used as the starting point of decoupled OM, it is fair to argue that OM would always outperform Prony-based methods by further minimizing the true model fitting error. This will be demonstrated by the simulation example in the next section.

4. Simulation Example

In this section, the performance of the optimal method is compared with that of a standard Prony-based algorithm using the FDTD-TS shown in Fig. 1, sampled at the output port of a microstrip two-port low-pass filter [7]. Both ports have a characteristic impedance of 50 ohms. The first 500 time-samples may contain transients, and are discarded. The samples between 501 to 1,500 are used for estimating the model, and the rest are used for prediction comparison. These 1,000 modeling samples are down-sampled (decimated) by a factor of 10:1 to 100 samples. The decimated FDTD-computed sequence is shown in Fig. 2, and is used next as the reference for comparison with the modeled sequence.

Fig. 3 shows the comparison for an ARMA(6,5) model (6-th order OM), and Fig. 4 compares the standard Prony's (also 6-th order) method used in [1]. In each case, the solid line corresponds to the decimated reference set, and the dashed line, to the model. The 6-th order model thus established has been then used to extrapolate the TS from 1,500 to 7,000 (*i.e.*, late-time data is extrapolated from an early-time model). Excellent comparison with the reference solution in Fig. 1 has been observed for the entire TS of 7,000 time steps. Prediction performance in the frequency-domain insertion loss (S_{21}) of the filter is shown in Fig. 5. The solid line corresponds to the measurements reported in [7], the dashed line to 6-th order OM, and the dot-dash line to 6-th order Prony. The filter is a notch low-pass filter, designed with a notch frequency of 7.5 GHz and attenuation of -40 dB. It is observed from Fig. 5 that OM outperforms Prony's by a significant margin, and corroborates excellently with measurements. In order to obtain a corroboration to the same accuracy, the Prony's model order has to be increased to 40 [1]. Therefore, OM is computationally much more efficient than Prony's in FDTD data processing. For higher-Q and electrically large structures, the computational savings will be much higher, because higher-order Prony's models can lead to severe instability. The return loss, S_{11}, also computed from the model, agrees very well with measurements. These results are omitted for brevity.

5. Conclusions

A classical ARMA identification technique has been used to model FDTD time sequences for high-Q structures. The major focus of the paper has been to demonstrate that, given the desired time-sample re-

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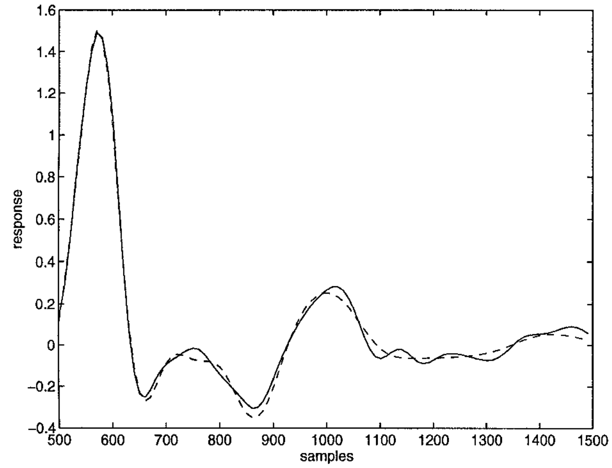

Figure 3: Comparison of the ARMA(6,5) model fit (dashed line) with the reference solution (solid line).

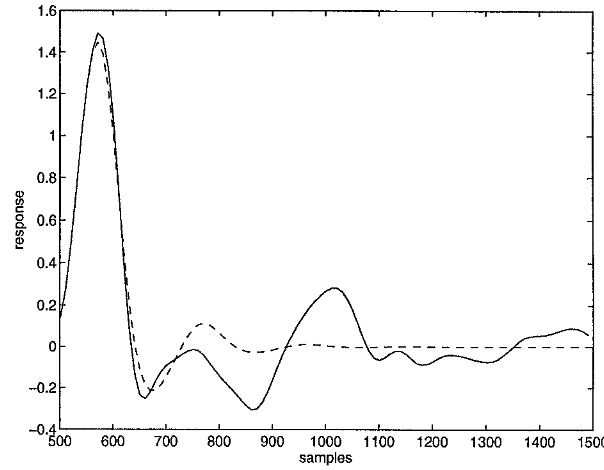

Figure 4: Comparison of Prony's 6-th order model fit (dashed line) with the reference solution (solid line).

sponse, it is possible to obtain accurate match in time and frequency domains with relatively lower model orders than hitherto reported in EM literature. It is shown that the multidimensional non-linear problem can be decoupled into two separate problems with reduced dimensionalities. The inherent mathematical structure of the non-linear denominator estimation problem is utilized in an efficient iterative computational algorithm. The new method has been shown to be very effective in the simulation of a microstrip filter.

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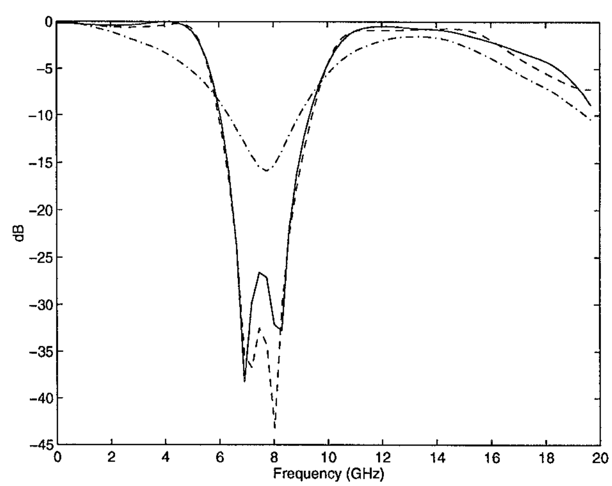

Figure 5: Comparison of frequency–domain insertion loss computed from extrapolated time sequences with measurements from [7].